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ANALYSIS OF A CYLINDRICAL SHELL VIBRATING IN A CYLINDRICAL FLUID REGION

by

Ho Chung, P. Turula, T. M. Mulcahy,
and J. A. Jendrzejczyk

BASE TECHNOLOGY

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Components Technology Division

August 1976

PREFACE

The work reported here was performed as part of the base-technology activity under the Flow-Induced Vibration Program (189a No. CA054-A) sponsored by ERDA/RRD. The overall objective of the activity is to develop new and/or improved analytical methods and guidelines for designing LMFBR components to avoid detrimental flow-induced vibration.

The thermal liner of the Fast Flux Test Facility (FFTF) reactor is a relatively thin cylindrical shell separated from the main reactor vessel by a narrow, fluid-filled annulus. The main function of the thermal liner is to shield the reactor vessel from excessive heat. Since the thermal liner is subjected to an environment of sodium flow, the potential for flow-induced vibrations must be assessed.

The present approach to the design of components subjected to flow includes separation of the structural natural frequencies from excitation frequencies carried by the fluid. However, generally accepted techniques that can be used to determine the free vibration characteristics of a thin shell surrounded by a narrow, fluid-filled annulus are not presently available to the reactor design engineer. This report describes an analytical and experimental study of a model similar to the thermal liner. A previously issued technical memorandum[1] contained a summary presentation of some of the work reported here. The present report provides a more complete compilation of the basic data and theory, together with a discussion of other research work in the subject area.

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NOMENCLATURE

Symbol	Description
f_n	Natural frequency
h, R, ℓ	Thickness, radius, and length of shell
K_R, K_θ, K_z	Elastic spring constant in radial, circumferential, and axial directions
m	Number of axial nodes
n	Number of circumferential waves
z, θ	Axial and circumferential coordinates
B	Bulk modulus of fluid
E	Young's modulus
ζ	Viscous damping factor
ν	Poisson's ratio
ρ	Mass density of shell
ρ_f	Mass density of fluid

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ABSTRACT

Analytical and experimental methods are presented for evaluating the vibration characteristics of cylindrical shells such as the thermal liner of the Fast Flux Test Facility (FFTF) reactor vessel. The NASTRAN computer program is used to calculate the natural frequencies, mode shapes, and response to a harmonic loading of a thin, circular cylindrical shell situated inside a fluid-filled rigid circular cylinder. Solutions in a vacuum are verified with an exact solution method and the SAP IV computer code. Comparisons between analysis and experiment are made, and the accuracy and utility of the fluid-solid interaction package of NASTRAN is assessed.

I. INTRODUCTION

The reactor vessel of the Fast Flux Test Facility (FFTF) is a heavy steel cylinder which holds the coolant and reactor internal structures. The thermal liner is a thin shell inside and concentric with the upper plenum of the reactor vessel. Liquid sodium from the primary coolant loops enters the reactor vessel near the lower end. Some inlet coolant is passed through the narrow annular region between the thermal liner and the reactor vessel to limit the probability of developing an excessive temperature gradient. Since the thermal liner is subjected to sodium flow, the potential for flow-induced vibrations must be assessed.

The Hydraulic Core Mockup (HCM) is a 0.286-scale model of the FFTF reactor vessel, including some reactor internals. The HCM was built to evaluate the hydraulic and vibrational characteristics of the FFTF reactor core and attendant components. Water is used to model sodium coolant because it has similar physical properties such as density and bulk modulus. The results of HCM analysis may be used to characterize the prototype FFTF through proper scaling laws and extrapolation.

The present report is an analysis of the vibration characteristics of a model of the thermal liner of HCM/FFTF. The analysis includes calculations and experimental evaluations of natural frequencies, mode shapes, modal damping, and response to harmonic forcing functions. The thermal liner was modeled as a thin, circular cylindrical shell with a fluid-filled annulus. The outer cylinder (reactor vessel) was assumed to be rigid, in consideration of its relative stiffness compared to that of the inner shell (thermal liner).

Since the early work of Arnold and Warburton,[2] the vibration problems of thin shells have been of interest to many structural engineers. Leissa[3] collected many results on vibrations of shells from published literature and summarized them in a unified manner. His report presents extensive frequency predictions for various boundary conditions and other complicating effects such as a surrounding fluid medium. Many authors[4-10] have investigated some natural-vibration characteristics for a cylindrical shell filled with fluid for various boundary conditions. The cylindrical shell with an annular fluid gap was first analyzed by Mnev[11,12] and recently by Levin and Milan,[13] Krajcinovic,[14] and Chen and Rosenberg.[15] However, these analyses were restricted to shells with simply supported boundary conditions and no axial constraint at the ends. This is the most amenable boundary condition in the analytical method designed to obtain a closed-form solution. The study presented here was undertaken to provide information for the cylindrical shell with a fluid-filled annulus rigidly clamped at its base and free at the top. This may be considered as a better representation of the boundary conditions of the thermal liner than simply supported conditions at both ends.

The following steps of analytical and experimental work were performed in order to characterize the dynamic behavior of the thermal-liner model:

1. Free-vibration analysis in vacuum (analytical and experimental studies).

2. Frequency response analysis in vacuum (analytical and experimental studies).

3. Time history analysis in vacuum (analytical study).

4. Free-vibration analysis for the thermal liner filled with water (analytical study).

5. Free-vibration analysis with a water-filled annulus (analytical and experimental studies).

In the analytical studies, case 1 was performed by using an exact solution method,[16] and two general-purpose finite-element computer programs: NASTRAN and SAP IV. Cases 2, 4, and 5 were carried out by using the NASTRAN computer program.[17] However, case 3 was done by the use of the SAP IV program.[18]

II. VIBRATIONS OF SHELLS

The modes of vibration of thin cylindrical shells are characterized by the number of circumferential waves (n) and axial nodes (m), as shown in Fig. 1. For each nodal configuration, three natural frequencies exist. The essential difference among these vibrations is the relative amplitudes of the motions in the axial, circumferential, and radial directions. Only the lowest frequency for each configuration is usually of practical importance because the other two are normally much higher in value.

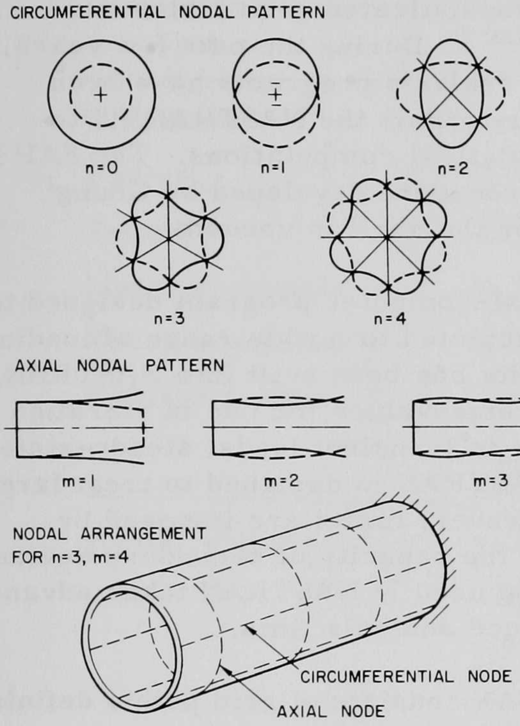

Fig. 1. Nodal Patterns of Clamped-Free Cylindrical Shell

For the modes having low n values, the dynamic behavior of a cylindrical shell is largely dependent upon the membrane stiffness of the shell. As the circumferential mode number increases, the bending stiffness also becomes important in determining the dynamic behavior of a cylindrical shell. For the modes having higher n values, the membrane stiffness is negligible and the bending stiffness is of prime importance. This phenomenon is generally true for any values of axial-node number (m). The lowest natural frequency is associated with the mode of the lowest total strain energy of the shell. Arnold and Warburton[2] first showed that the lowest natural frequency frequently corresponds to a mode having a higher n value rather than the modes having low n values such as n = 0, 1.

Forsberg[19] investigated the influence of boundary conditions on the modal characteristics of thin cylindrical shells for a wide variety of boundary conditions. However, a reasonable analytical representation of actual boundary conditions still remains in question. There are a number of efforts[20,21] to clarify the frequent discrepancies between analytical predictions and experimental observations of shell natural frequencies, especially for the modes associated with large membrane stiffness of the shell. These discrepancies are often attributed to imperfect boundary support,[21] nonlinear behavior of the shell,[20] and different levels of modal dampings. As described in this report, an effort was made to bracket the experimentally observed natural frequencies by analytical predictions for a few boundary conditions that could reasonably simulate the experimental setup. In addition, frequency response analysis was performed to compare the level of output for different modes of vibration in order to investigate the difficulty in finding the low-n-value modes in the experiment.

III. ANALYTICAL METHODS

Free-vibration problems for cylindrical shells in a vacuum have been solved by many contributors through many different ways. Exact solutions were obtained by Forsberg,[19] Warburton,[22] and Chung.[16] Utilizing the finite-element method, many shell problems with sophisticated geometries and physical properties have been also treated.[9,23-25] During the past few years, a number of general-purpose finite-element analysis programs have been developed and widely used in industry. In this report the NASTRAN finite-element analysis program[17] was used for analytical computations. The SAP IV finite-element code[18] and an exact solution procedure developed by Chung[16] were also used as corroborating solutions for the shell in vacuum.

NASTRAN is a general-purpose digital-computer program designed to analyze the behavior of elastic structures subjected to a wide range of loading conditions. Considerable analytical versatility has been built into NASTRAN, including determination of real and complex eigenvalues for use in vibration analysis, and evaluation of dynamic response to transient loads, steady-state harmonic loads, and random excitations. NASTRAN is designed to treat large problems with many degrees of freedom; however, limits are imposed by practical considerations of running time and the capacity of secondary devices used by the computer. The numerical method used in NASTRAN takes advantage of matrix sparsity and bandwidth to reduce analysis time.

The structural model used in NASTRAN consists of grid points defining degrees of freedom, structural elements connected between grid points, and loads applied to grid points. NASTRAN has a wide variety of elements and constraint between degrees of freedom. One of the unique features of NASTRAN is the fluid-elastic analysis capability. The program can perform free-vibration analysis, frequency response, and transient analysis for an axisymmetric (geometrically) structure containing fluid. The compressibility and gravity effect of a free surface can be included in the analysis. The motions of the fluid are assumed to be small compared to the size of the container.

The SAP IV finite-element code has similar capabilities, but its analysis versatility is limited compared to NASTRAN. However, the SAP IV program was found to be more economical for problems analyzed here in terms of computer run-time.

An exact solution method[16] also was used to evaluate natural frequencies of the thermal liner in vacuum. The solution was obtained from a direct-solution procedure of Sanders' shell equations in which the modal displacement functions were constructed from simple Fourier-series expressions.

The dimensions of the shell considered in this report were scaled from the thermal liner of the FFTF reactor. Several fluid-annulus sizes were considered. The scale factor is about 1/14, giving approximate model dimensions of 20.5-in. (52.1-cm) height, 17-in. (43.2-cm) diameter, and 0.039-in. (0.098-cm) thickness. The geometrical, as well as material, properties of the scaled shell model of the thermal liner are given in Table I.

TABLE I. Properties of Shell Model of Thermal Liner

Inner Shell	
Thickness	h = 0.058 in. (0.15 cm)
Radius	R = 8.510 in. (21.62 cm)
Length	ℓ = 20.125 in. (51.12 cm)
Material density	ρ = 701 x 10^{-6} lb-sec^2/in.^{4} (7492 kg/m^3)
Young's modulus	E = 26.5 x 10^6 psi (1.83 x 10^{11} Pa)
Poisson's ratio	ν = 0.3
Water Annulus	
Radial-gap size	0.151 in. (0.38 cm)
	0.253 in. (0.64 cm)
	0.538 in. (1.37 cm)
	1.033 in. (2.62 cm)
	2.94 in. (7.47 cm)
Fluid density	ρ_f = 93.6 x 10^{-6} lb-sec^2/in.^{4} (1000 kg/m^3)
Bulk modulus	B = 300,000 psi (2.07 x 10^9 Pa)
Outer Shell	
Assumed rigid	

IV. EXPERIMENTAL INVESTIGATIONS

A. Thermal-liner Model

The experimental model of the thermal liner was fabricated by rolling a steel plate, seam-welding it, and soldering it to a 0.5-in.-thick (1.27-cm) brass plate which was mounted to a 1-in.-thick (2.54-cm) steel plate. This steel plate was then bolted to a heavy steel base block. To test the thermal-liner model with different water-gap sizes, several outer cylinders were made. To obtain a uniform gap, in spite of slight out-of-roundness or imperfections, these cylinders were formed by casting relatively thick (~2 in.) (5.1 cm) concrete around the steel test cylinder using hard durometer neoprene spacers to obtain the desired spacing. The concrete was water-proofed and attached to the steel base block as shown in Fig. 2.

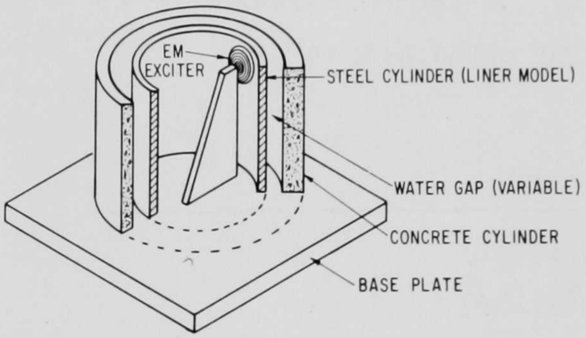

Fig. 2. Schematic of Thermal-liner Model. ANL Neg. No. 113-5732 Rev. 1.

The average diameter of the fabricated shell was determined by taking the average of 36 measurements. The deviation of these measurements was less than 0.5%. The annular gap size was determined by measuring the volume of water necessary to fill the annular region. For any of the gap sizes, the maximum deviation from the average values was about 10%. (The geometric accuracy of the model may exceed the uniformity expected in a prototype thermal liner.) The density and elastic modulus of the thermal-liner model were determined by weighing and by performing cantilevered-beam frequency tests of strips of steel plate used in fabricating the shell. The dimensions and material properties of the thermal-liner model have been given in Table I.

The inner steel cylinder was excited by an electromagnet positioned inside the shell (see Fig. 3). The exciter coil (280 turns of No. 16 enamel-covered wire) was mounted on a heavy steel support column, and had a driving area of about 26 in.^{2} (0.017 m^2). The gap between magnet and steel cylinder was about 3/16 in. (0.48 cm). The magnet drive current was provided from an audioamplifier (McIntosh Model MI-200AB, 200 W) controlled by signal generators (Hewlett-Packard Model 203A Variable Phase Generator, and Model HOI-3722A Noise Generator). Both sinusoidal and wide-band random signals were generated.

Motion of the top of the steel cylinder was monitored by seven miniature piezoelectric accelerometers (Endevco Picomin No. 22) cemented to the inside top surface of the test cylinder every 30°, with 0° defined to be opposite the center of the magnet. In addition, three accelerometers were mounted along

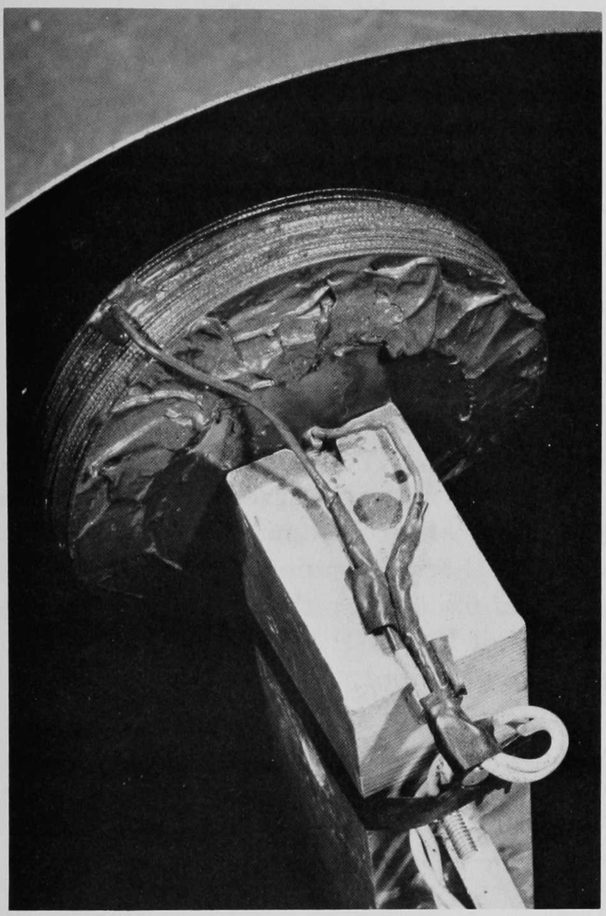

Fig. 3. Electromagnetic Exciter.
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the 0° longitudinal line on the cylinder to examine the longitudinal-mode shape. A movable accelerometer mounted on the magnet was used to search for circumferential nodal points for the modes having higher n values. These accelerometers were connected through charge amplifiers to a patch panel for ease in monitoring the signals.

B. Test Procedure

The test procedure consisted of three phases. First, natural frequencies were determined by exciting the shell with a wide-band random force and inspecting power-spectral-density plots produced by a Fourier analyzer (Hewlett-Packard Model 5451/A) from the time-history signals of several accelerometers. Second, the shell was excited again with a sinusoidal current supplied to the coil, using a range of frequencies in the vicinity of each of the natural vibration frequencies detected by random excitation. For each natural frequency f_n, the accelerometer signal in a narrow band about this frequency was processed by the Fourier analyzer to provide a more accurate value of the peak frequency and to establish the rms acceleration at each accelerometer position. This information was plotted to identify mode shapes corresponding to the natural frequencies. In the third phase of testing, the transfer function between the rms displacement and peak coil current was plotted at discrete points in a narrow band about each natural frequency so that the equivalent viscous damping ratio could be calculated by the half-power-point bandwidth method.

V. DISCUSSION OF RESULTS

A. Vibration in Vacuum

Before considering the shell vibrating in a fluid, we studied the shell in vacuum to establish the significance of variations in the boundary conditions and the degree of correspondence to be expected between analytical and experimental results.

1. Natural Frequencies

The natural frequencies for the thermal liner in vacuum were calculated by an exact method, NASTRAN, and SAP IV. The finite-element model used in the computer codes was the same and consisted of 10 divisions vertically and 9 divisions over a quarter of the shell circumferentially. These analytical predictions are compared in Table II for clamped-free boundary conditions. NASTRAN gave frequencies up to 5% higher than those obtained by the exact-solution method. The solutions of the SAP IV code are much closer to the exact solutions (less than 2% deviation).

TABLE II. Comparison of Calculated Natural Frequencies (in Hz) for Shell in Vacuum with Clamped-Free Boundary Conditions

n	m	Exact Method	SAP IV	NASTRAN
1	1	855.10	-	856.3
2	1	403.72	405.1	410.1
3	1	223.34	225.6	232.2
4	1	171.77	174.3	180.5
5	1	199.16	201.7	206.2
6	1	268.86	272.0	275.5
7	1	361.92	366.2	370.1
8	1	472.54	478.4	483.5
9	1	599.03	-	614.0
3	2	928.28	-	943.2
4	2	644.48	-	671.4
5	2	494.69	505.8	529.3
6	2	442.00	454.3	478.0
7	2	464.59	476.7	496.9
8	2	539.45	-	567.2
9	2	648.34	-	673.2

Although the experimental setup was intended to simulate a fixed condition at the base, complete rigidity of axial and rotational restraint was not achieved. Thus the base boundary conditions should be properly modeled in the analytical method to correlate with the experimental results. Five different cases of bottom boundary conditions were considered:

(1) Clamped (C)

(2) Simply supported (SS)

(3) Spring supported (Sp): $K_R = K_\theta = 10^6$, $K_z = 10^6$

(4) Spring supported (Sp): $K_R = K_\theta = 10^6$, $K_z = 10^5$

(5) Spring supported (Sp): $K_R = K_\theta = 10^6$, $K_z = 10$.

Here the unit of spring restraint is lb/in. per 1.4844 in. (4.645 kN/m per m) of shell circumference. In the clamped condition, all displacements and rotations were restrained; in the simply supported condition, only displacements were restrained. In the NASTRAN model, the base boundary condition was further relaxed by allowing movement in axial, circumferential, and radial directions against elastic springs at the grid points (see Fig. 4).

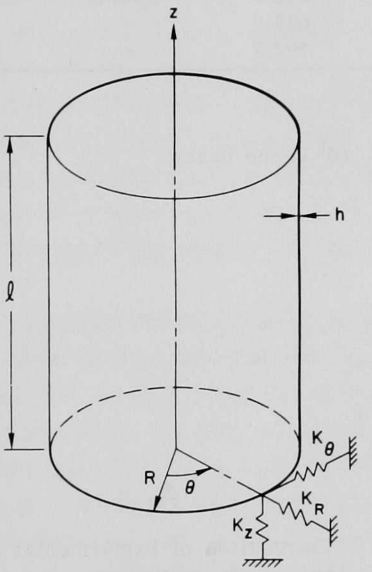

Fig. 4

Elastic-spring-supported Shell

Table III and Fig. 5 compare the experimental results for the shell vibrating in air (the effect of air is often neglected) to the analytical predictions of the NASTRAN program for different bottom boundary conditions. In comparison with the analytical predictions for the clamped-free shell, the experimental results are smaller for the modes having lower n values and larger for those having higher n values. This may be due to some degree of eccentricity[3] in the experimental model and errors in evaluating its physical properties. Although not distinguishable in Fig. 5, the natural frequencies are higher (<1%) for the clamped base condition than for the simply supported base condition. Thus it can be concluded that rotational restraint at the base does not significantly affect the overall results and is primarily related to a local flexure condition.

TABLE III. Comparison of Calculated Natural Frequencies (in Hz) for Shell in Vacuum with Experimental Values in Air for Various Support Conditions

		NASTRAN Predicitions					
				Sp-F[c]			Experiment in Air
n	m	C-F[a]	SS-F[b]	$K_z = 10^6$	$K_z = 10^5$	$K_z = 10$	
0	1	-	-	-	-	14.5	-
1	1	856.3	856.2	782.5	525.8	-	-
2	1	410.1	406.7	357.5	204.7	21.9	-
3	1	232.2	228.8	198.1	117.4	-	-
4	1	180.5	180.4	162.6	127.6	112.5	152
5	1	206.2	204.9	199.3	185.4	-	202.2
6	1	275.5	275.5	271.8	266.6	265.0	292.7
7	1	370.1	366.6	365.6	365.5	-	399
8	1	483.5	483.5	482.3	480.9	480.5	520.8
9	1	614.0	613.7	613.2	612.3	-	665
10	1	760.6	760.6	760.0	759.4	759.3	815
3	2	943.2	-	895.2	860.9	-	-
4	2	671.4	670.9	628.5	585.9	573.1	-
5	2	529.3	527.7	492.6	455.6	-	461
6	2	478.0	477.3	449.3	422.4	415.7	437
7	2	496.9	496.2	476.3	458.8	-	488.9
8	2	567.2	566.7	552.9	542.0	539.6	580.6
9	2	673.2	673.2	663.1	655.9	-	710
10	2	804.9	804.5	797.4	792.7	-	820

[a]C-F: Clamped-Free.
[b]SS-F: Simply supported-Free.
[c]Sp-F: Spring-supported-Free ($K_R = K_\theta = 10^6$ at the base).

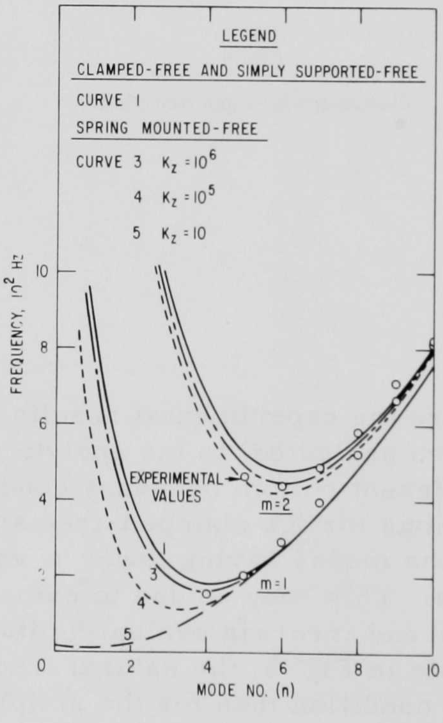

Fig. 5

Comparison of Experimental and NASTRAN-predicted Natural Frequencies for Shell without Fluid. ANL Neg. No. 113-5728 Rev. 1.

In contrast, the influence of axial constraint (K_z) is evident and significant throughout most of the region of interest. The natural frequencies become much smaller as the axial constraint (K_z) is relaxed. The lowest natural frequencies for shell-type vibration ($n \geq 2$) occur at $n = 2$ for $K_z = 10$, in contrast to $n = 4$ for $K_z = 10^6$. For higher axial half-waves ($m = 2$), the differences are much smaller throughout the entire region of circumferential wave number n. The experimental results are somewhat closer to the case of spring-supported base condition of $K_z = 10^6$ for higher circumferential wave numbers ($n \geq 4$).

All experimental efforts failed to detect the response of modes associated with lower circumferential wave numbers ($n < 4$). In addition to electromagnetic excitation, these efforts included acoustic excitation at various amplitude levels by a loud speaker. Fourier analysis of the acceleration response to a random excitation did show a response at 140 Hz, which probably corresponds to the $n = 3$, $m = 1$ mode, but there was no corresponding response when a single harmonic excitation at this frequency was generated.

Other investigators[2,20-22] have also reported difficulty in obtaining natural frequencies for the low-n-value modes. However, the explanations posed here and elsewhere are not totally satisfactory. The frequencies of the low-n-value modes are highly dependent on the stiffness K_z, as shown in Fig. 5, so that the relatively undeterminable and possibly nonlinear nature of this restraint may greatly reduce the sharpness of the associated response. However, for the modes that gave a clear response, increasing the driving force by a factor of 4 lowered the peak frequency by at most 0.5%; hence, nonlinearity of support appears to be negligible.

One explanation is in the strain-energy consideration:[20,22] The low-n-value modes associated with larger membrane energy may be difficult to excite, compared to the higher-n-value modes with larger bending energy. Also, the difficulty may be the masking effect of the higher-n-value modes at frequencies close to that of a low-n-value mode. This is particularly suspected in the tests using electromagnetic excitation; when a pure harmonic current is applied to the device, the resulting force function carried higher harmonic frequencies of an amplitude up to 20% of the fundamental harmonic. Finally, geometrical imperfections in the as-fabricated shell are a factor that should not be ruled out of consideration.

2. Frequency Response Analysis

The steady-state response to a radial harmonic force was predicted by NASTRAN for a thermal-liner model with two different boundary conditions: (1) clamped-free, and (2) spring-supported ($K_z = 10^6$)-free. (In the previous section, the experimentally determined natural frequencies were found to approximate the case of spring-supported–free shell.) The latter boundary conditions were simulated in an effort to identify the difficulty in exciting the low-n-value modes. In operation, the driving force was applied at one point at the top of the shell and the responses were calculated at selected points on the shell surface. The particular response of interest was the acceleration due to a sinusoidal force with 1 lb (4.45 N) maximum amplitude, the so-called acceleration mobility. The analysis was based on the normal-mode method, with the lowest 35 normal modes and an assumption of 2% structural damping.

Figures 6 and 7 show the spectrum of acceleration mobility for three selected points to a loading at the top of the shell: (a) the same point as the load point, (b) a point 90° from the load point, and (c) a point diametrically opposite the load point. As evidenced by the figures, the acceleration responses corresponding to modes (n = 2, 3) with low circumferential wave number are smaller than those with higher n for both cases of shell boundary conditions. Also, the modes (n, m = 8, 1; 6, 2 in Fig. 6; and n, m = 8, 1; 5, 2; 7, 2 in Fig. 7), having nearly equal natural frequencies, were superimposed, and the response is much larger than the others. Consequently, it is difficult to isolate the component natural frequencies.

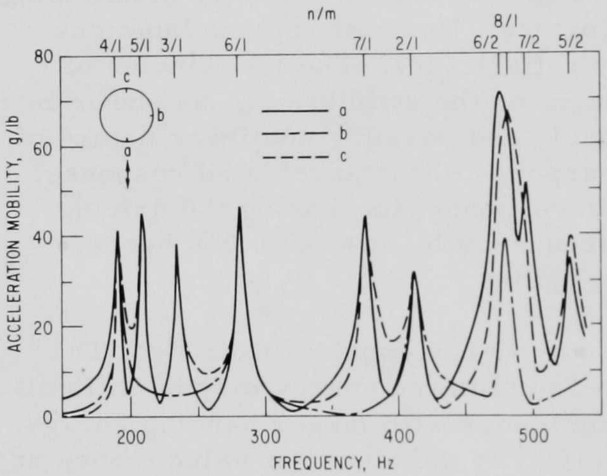

Fig. 6. Acceleration Mobility at Selected Points due to Harmonic Loading at Point "a" for a Clamped-Free Shell

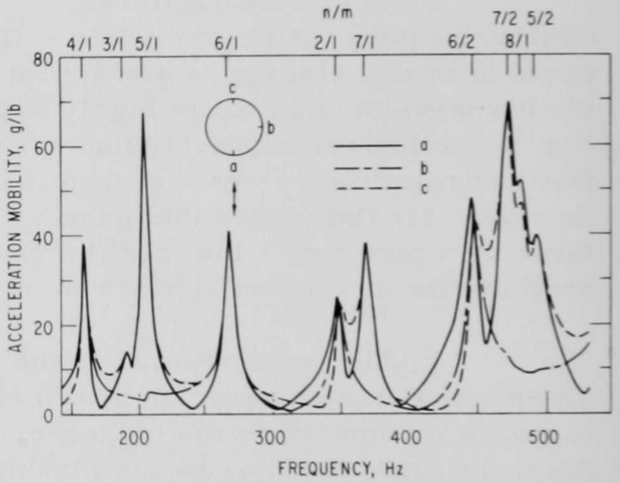

Fig. 7. Acceleration Mobility at Selected Points due to Harmonic Loading at Point "a" for a Spring-supported ($K_R = K_\theta = K_z = 10^6$)–Free Shell

Figures 6 and 7 also show that the response at point "b" does not experience the resonance of the modes with odd n values (n = 3, 5, 7, ...) because this point falls on a node line.

3. Time History Analysis

When the natural frequencies for different shell-vibration modes approximate each other, it may be difficult to excite one particular mode by a sinusoidal force. This may produce a situation in which these modes are coupled together, with consequent difficulties in identifying the mode shape for a particular resonance. An analytical study was conducted, using the SAP IV computer program, to demonstrate this aspect of shell-vibration phenomena.

The time history analysis was performed for a clamped-free shell due to a sinusoidal force. The force was applied in radial direction on a small area (rather than at a point) near the top of the shell. First, the shell was excited with the natural frequency corresponding to the n = 3, m = 1 mode. Examination of the resulting time-history data showed the shell was vibrating with the n = 3, m = 1 mode. In Fig. 6, frequency response analysis showed this mode was distinctly identified and not close to other modes. However, when the shell was excited with the natural frequency of the n = 8, m = 1 mode, the resulting shell motion was a coupled one of the n = 8, m = 1 mode and the n = 6, m = 2 mode. Therefore, in all experiments an extremely careful discrimination is essential to accurately identify the mode shapes and not to miss any natural frequencies.

B. Effect of Inner Fluid

Vibrations of a cylindrical shell containing a liquid have been studied extensively.[3-12] Since the fluid has a considerable inertial effect on the shell, the natural frequencies are reduced significantly.

Free vibrations of the thermal-liner model filled with fluid were analyzed, using the NASTRAN program. The support condition for this thermal liner was assumed as clamped-free. The finite-element model consisted of 10 vertical divisions and 15 circumferential divisions over a quarter of the shell, and 10 divisions radially through the inside fluid. First, NASTRAN solutions for other shell geometries were evaluated by comparing them to the available analytical closed-form solutions.[5-8, 10-12] For a variety of shell geometries and two cases of boundary conditions (clamped-free and simply supported without axial constraints), NASTRAN yielded satisfactory predictions of natural frequencies.

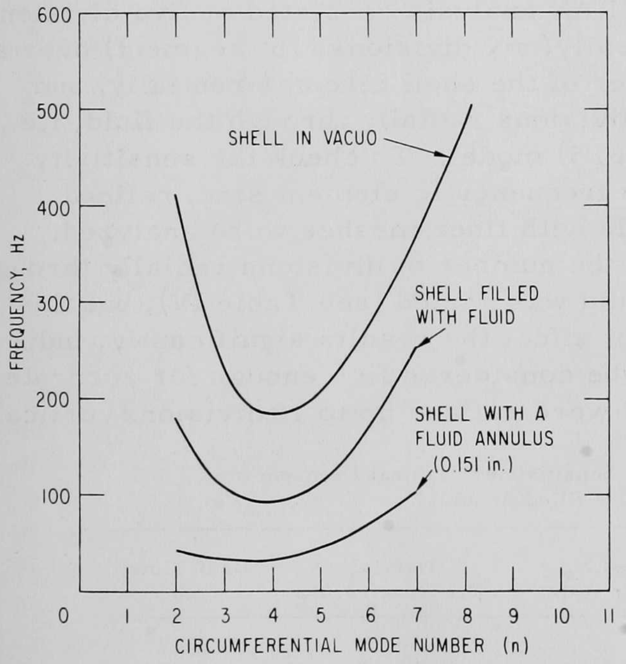

Fig. 8. Natural Frequencies for Shell Filled with Fluid (m = 1)

Figure 8 shows the natural frequencies for the thermal-liner model filled with fluid. Also shown are natural frequencies for the thermal-liner model in vacuum and with a fluid-filled annulus only. For modes n = 2-7 and m = 1, the inside fluid of the shell lowered the natural frequencies by the factor of 1.4-2.3; however, with a fluid-filled annulus of 0.151 in. (0.38 cm), the modes were lowered by a factor of 3.6-9.5. These factors become smaller as the circumferential wave number n increases for both cases of the inside fluid and the fluid-filled annulus. Natural frequencies for the shell with different size of fluid-filled annulus are discussed in Sec. C below.

C. Effect of Fluid Annulus

The natural frequencies of the shell with a fluid-filled annulus were studied by three different ways: an experimental investigation, a simple added mass correction[15] to the solutions in air, and a NASTRAN program analysis. Figure 9 summarizes the results.

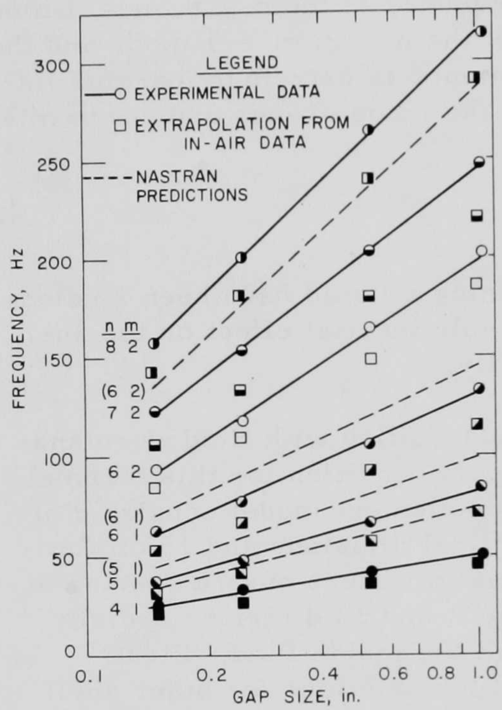

Fig. 9

Experimental and Predicted Natural Frequencies for Shell with a Fluid-filled Annulus. ANL Neg. No. 113-5730 Rev. 1.

The added mass corrections due to the fluid annulus were made with the experimental data for the shell vibrating in air. The analytical evaluation of the added mass factors was made by assuming the shell was infinitely long. A comparison between the experimental results and these extrapolations shows this method appears to give frequency predictions that are somewhat low, but generally within 10% of the experimental results.

The finite-element model for the NASTRAN analysis consisted of five divisions vertically, six divisions (15° segment) over a quarter of the shell circumferentially, and five divisions radially through the fluid, i.e., a (5, 6, 5) model. To check the sensitivity of the frequency to element size, refined models with finer meshes were analyzed. First the number of divisions radially through the fluid was varied (see Table IV), but this did not affect the results significantly, indicating that the five-division model can be considered fine enough for accurate results. Then the meshes for the shell were refined up to 10 divisions vertically

TABLE IV. Finite-element Size Sensitivity of Natural Frequency for Shell with Fluid-filled Annulus[a]

No. of Divisions for Shell		No. of Radial Divisions for Water	Natural Frequencies, Hz	CPU Time,[b] sec
Vertical	Circumferential			
5	6	1	78.48	47
5	6	2	77.97	47
5	6	5	77.82	49
5	6	8	77.81	51
5	6	5	77.82	49
10	6	5	71.65	176
10	9	5	67.69	274
10	15	5	65.85	498

[a]Clamped-free shell with 1.033-in. fluid annulus ($n = 4$, $m = 1$).
[b]NASTRAN level 15.1 was run on IBM S/370-195. CPU time was spent to evaluate one frequency.

and 15 divisions circumferentially (6° segment). This (10, 15, 5) model resulted in a 20% lower natural frequency. However, the cost of running a series of fluid-solid interaction problems with NASTRAN, particularly with a much finer mesh than used here, would be prohibitive. To find one natural frequency, the runs involving (5, 6, 5) mesh required about 1 min of IBM S/370-195 CPU time; the (10, 15, 5) case required about 8 min. Both were run using the NASTRAN level 15.1.

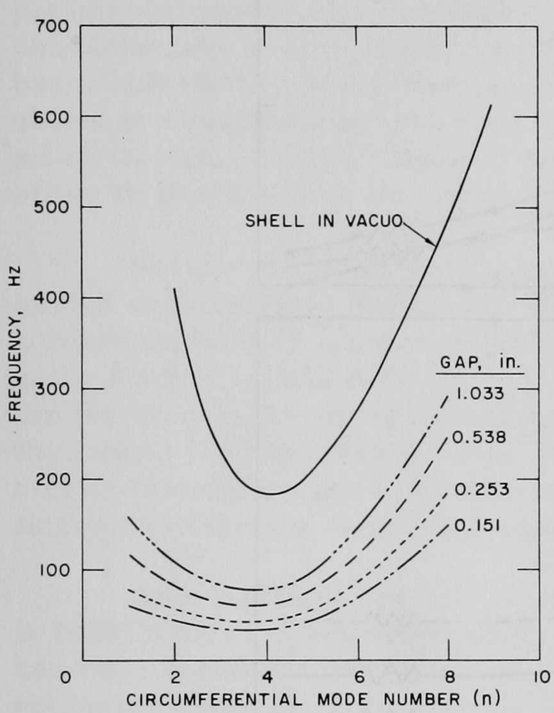

Fig. 10. NASTRAN-predicted Natural Frequencies for Shell with a Fluid-filled Annulus (m = 1). ANL Neg. No. 113-5731.

Figure 10 shows the natural frequencies predicted by the NASTRAN program for the clamped-free shell with four different fluid gaps. Natural frequencies for the shell in vacuum are also shown for comparison. As the gap size decreases, the frequency becomes smaller for the entire region of circumferential wave number n. These NASTRAN-predicted frequencies are also compared to other results in Fig. 9. The NASTRAN predictions are 20-30% higher than the experimental results. This difference may be narrowed by using the finite-element model with finer meshes. However, one may still expect the uncertain shell-boundary condition (see Sec. A above) to cause some discrepancies between analytical predictions and experimental results.

D. Damping

For purposes of determining the equivalent viscous damping ratios, the frequency-response curve was first determined in the 0.1-0.5-g acceleration range and then redetermined at the highest acceleration level compatible with the available equipment. The ratio of acceleration levels was at least two and usually five to ten. The damping-ratio variation with amplitude level was in the 10-40% range. Figure 11 shows selected average values of damping for the shell vibrating with various water gaps and in air. Clearly, the smaller fluid gaps are associated with higher damping ratios than those measured in air, typically twice as large. Generally, larger damping is associated with smaller water gaps. For those few cases that do not follow this later trend, the frequency-response curves probably were distorted (broadened) by superposition of the response corresponding to an adjacent natural frequency which could not be accounted for in the half-power bandwidth computation.

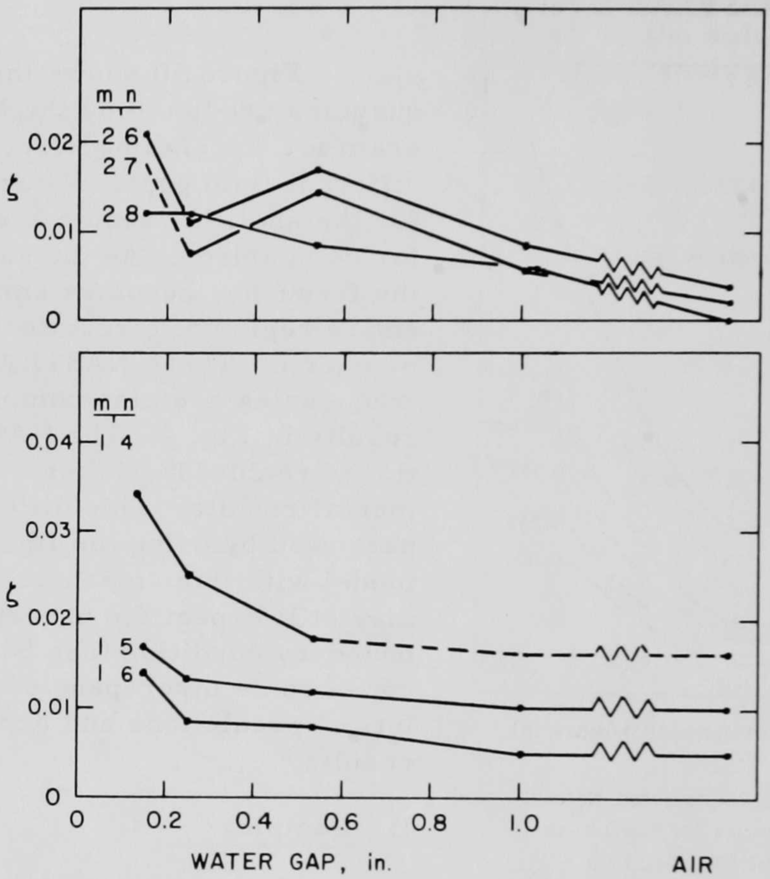

Fig. 11. Experimentally Determined Damping Ratios.
ANL Neg. No. 113-5809 Rev. 1.

VI. CONCLUSIONS

Vibration-analysis methods for shells, such as the thermal liner of the FFTF reactor, are illustrated and comparisons were made with experimental results from a scale model of the thermal liner. Correspondence of experimental and analytical results is within acceptable limits for design purposes once the shell-boundary conditions are properly modeled. The experimental model was designed to be a fixed-free shell; however, its modal characteristics were similar to the shell with elastic spring support at the base. Flexibility of the base support in the axial direction of the shell was shown to considerably lower the natural frequencies, especially for the small-n-value modes. Natural frequencies were shown to be rather insensitive to flexibility of the base support in the radial and tangential directions.

Severe vibration modes corresponding to solutions with low n values eluded experimental detection. Efforts to resolve this experimental difficulty included a frequency-response analysis and a time history analysis using NASTRAN and SAP IV. The efforts revealed that acceleration mobilities for the membrane energy dominant modes (small-n-values modes) were lower than those for the other modes. However, further investigation is required to clarify the significance of these modes in design and the reasons for the difficulty in detecting them experimentally.

The feasibility of using coupled fluid-elastic finite-element analysis in NASTRAN to solve vibration problems involving shells in fluid medium has been demonstrated; however, the computer time costs incurred prohibit extensive application.

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